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Matrix Coefficients of the Principal P_J-series and the Middle Discrete Series of $SU(2,2)$

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§0. Motivation and introduction

Recently several people including the authors investigate various kinds of generalized spherical functions on relatively small Lie groups. The typical situation of the problem is as follows.

Let G be a semisimple Lie group with finite center and R a closed subgroup. Let π be an admissible (irreducible) representation of G, and η an irreducible admissible or unitary representation of R. Form the smooth induction of $\mathrm{Ind}_R^G(\eta)$ from R to G. Then we consider the intertwining space $\mathrm{Hom}_G(\pi, \mathrm{Ind}_R^G(\eta))$.

Our concern is the case where R is "large enough" so that the above intertwining space becomes of finite dimension. This kind of problem is standard in the local theory of automorphic forms and representations.

If we take a nonzero element $T \in \mathrm{Hom}_G(\pi, \mathrm{Ind}_R^G(\eta))$, then the image $\mathrm{Im}(T)$ of T is a realization (or a model) of π in $\mathrm{Ind}_R^G(\eta)$.

A most popular case is when R is the maximal unipotent subgroup and η a unitary non-degenerate character of R. Then the intertwiners are called Whittaker functionals. These are investigated mainly to define automorphic L-functions in GL_n case (Jacquet, Piatetski-Shapiro and Shalika [8, 9]) not only for the real field but also for the p-adic fields. Fundamental results on Whittaker models over the real field are found in the works of Kostant [11], Vogan [24] and Matumoto [12, 13].

More generally, there is a notion of Gel'fand-Graev representation, where R is a "good" subgroup of some parabolic subgroup of G. An attempt to classify representation of G using this kind of realization in $\mathrm{Ind}_R^G(\eta)$ is worked out by Yamashita [26, 27] for $G = SU(2,2)$.

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