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Characters of Non-Linear Groups

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§1. Introduction

Two of the primary methods of constructing automorphic forms are the Langlands program and Howe's theory of dual pairs.

The Langlands program concerns a reductive *linear* group G defined over a number field. Associated to G is its dual group ${}^LG$. The conjectural principle of functoriality says that a homomorphism ${}^LH \to {}^LG$ should provide a "transfer" of automorphic representations from H to those of G.

On the other hand Howe's theory of dual pairs, the theta correspondence, starts with the oscillator representation of the *non-linear* metaplectic group $Mp(2n)$, the two-fold cover of $Sp(2n)$. Restricting this automorphic representation to a commuting pair of subgroups (G, G') of $Mp(2n)$ gives a relationship between the automorphic representations of G and G'.

This suggests a natural question: is the theta-correspondence in some sense "functorial". As Langlands points out [16]: "the connection between theta series and functoriality is quite delicate, and therefore quite fascinating ... ". Now G and G' may be non-linear groups, and so even to define the notion of functoriality requires some work. In particular the L-groups of G and G' are not defined. Nevertheless it is reasonable to ask that theta-lifting be given by some sort of data on the "dual" side. This can be done in some cases in which the non-linearity of G and G' do not play an essential role. Nevertheless a proper understanding of the relationship between theta-lifting (and its generalizations) and functoriality requires bringing the representation theory of non-linear groups into the Langlands program.

Some discussion of the relation of the theta-correspondence to functoriality may be found in [15], [21], and [2]. The case of $U(3)$ has been

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