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A Method of Prolongation of Tangential Cauchy-Riemann Equations

Chong-Kyu Han and Jae-Nyun Yoo*

§0. Introduction

In this paper we present a method of prolongation of tangential Cauchy-Riemann equations. The technique is, roughly speaking, separating the holomorphic derivatives of CR functions from their complex conjugates and applying the tangential Cauchy-Riemann operators to the holomorphic part. Using this method we show that under generic assumptions mappings of a CR manifold into a CR manifold of higher dimension satisfy a certain Pfaffian system in the jet space, which implies the rigidity and the regularity of CR mappings.

Let M be a differentiable manifold of dimension $2m+1$. A CR structure on M is a subbundle $\mathcal{V}$ of the complexified tangent bundle $T_{\mathbb{C}}M$ having the following properties:

i) each fiber is of complex dimension m,

ii) $\mathcal{V} \cap \bar{\mathcal{V}} = \{ 0 \}$,

iii) $[\mathcal{V}, \mathcal{V}] \subset \mathcal{V}$ (integrability).

It is well known that if $(M, \mathcal{V})$ is real analytic (C^ω) M is locally embeddable into $\mathbb{C}^{m+1}$ as a real hypersurface. In this paper we are concerned with CR mappings of M into a C^ω real hypersurface N of $\mathbb{C}^{n+1}$, $n \geq m$. Let N be a C^ω real hypersurface of nondegenerate Levi form in $\mathbb{C}^{n+1}$ defined by $r(z,\bar{z}) = 0$, where $z = (z_1, \cdots, z_{n+1})$. Let A and B be $(n+1)$-tuple of nonnegative integers and let $z^A = z_1^{a_1} \cdots z_{n+1}^{a_{n+1}}$ if $A = (a_1, \cdots, a_{n+1})$. After a holomorphic change of coordinates $r(z,\bar{z})$ can be written as

$$r(z,\bar{z}) = z_{n+1} + \bar{z}_{n+1} + \sum_{j=1}^{n} \lambda_j z_j \bar{z}_j + \sum_{A,B} c_{AB} z^A \bar{z}^B, \tag{0.1}$$

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