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The Infinitesimal Spectral Rigidity of the Real Grassmannians of Rank Two

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Dedicated to Professor M. Kuranishi on his 70th birthday

Introduction

Let (X, g) be a Riemannian symmetric space of compact type. Consider a family of Riemannian metrics $\{g_t\}$ on X, for $|t| < \varepsilon$, with $g_0 = g$. We say that $\{g_t\}$ is an isospectral deformation of g if the spectrum of the Laplacian of the metric g_t is independent of t. We say that the space (X, g) is infinitesimally spectrally rigid (i.e., spectrally rigid to first order) if, for every such isospectral deformation $\{g_t\}$ of g, there is a one-parameter family of diffeomorphisms φ_t of X such that $g_t = \varphi_t^* g$ to first order in t at $t = 0$, or equivalently if the infinitesimal deformation $\frac{d}{dt} g_{t|t=0}$ of $\{g_t\}$ is a Lie derivative of the metric g.

In [13], V. Guillemin proves that the infinitesimal deformation h of an isospectral deformation of g satisfies the following integral condition: for every maximal flat totally geodesic torus Z contained in X and for all parallel vector fields ζ on Z, the integral

$$\int_Z h(\zeta, \zeta)\, dZ$$

vanishes, where dZ is the Riemannian measure of Z. If all of these integrals corresponding to a symmetric 2-form h on X vanish, we say that h satisfies the Guillemin condition. It is easily verified that a Lie derivative of the metric always satisfies the Guillemin condition. We say that the space (X, g) is rigid in the sense of Guillemin if the following Radon transform property holds on X: the only symmetric 2-forms on X satisfying the Guillemin condition are the Lie derivatives of the metric g. Thus according to [13], if the space (X, g) is rigid in the sense of Guillemin, it is infinitesimally spectrally rigid.

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