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# Vector-Valued Forms and CR Geometry

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***Dedicated to Professor M. Kuranishi on his 70th birthday***

## §1. Introduction.

Vector-valued forms arise in the study of various higher codimensional geometries. This note gives an overview of how the invariant theory of the Levi form (a vector-valued form) can be used to understand higher codimensional $CR$- structures.

Roughly speaking, the Levi form of a $CR$- structure of codimension $c$ on a manifold $M$ of dimension $2n+c$ can be interpreted as a map from $M$ to the vector space consisting of $c$-tuples of $n \times n$ hermitian matrices (a vector space that we denote as Herm). However, this interpretation depends on a prior choice of moving coframesthat is, local sections of the cotangent bundle of $M$. Fortunately, there is a natural action of the group $G = GL(n, C) \times GL(c, R)$ on Herm that accounts for the effects of these choices. More precisely, there is a natural map from $M$ to the quotient space Herm$/G$. Knowledge about the structure of this quotient space can be used to define canonical objects in higher codimensional $CR$- geometry. At present, the best developed example (discussed in §5) is a canonical connection for suitably generic $CR$- structures. The simplest examples, though, are functions defined on Herm$/G$, or equivalently, $G$-invariant functions defined on Herm : these lead one to explore the invariant theory of vector-valued forms as a tool in the study of $CR$-geometry. The history of invariant theory suggests two lines of approach. The first, discussed in §3, is to use methods of classical invariant theory to find explicit polynomial functions of vector-valued forms that are (relatively) invariant under the group action. While these techniques are quite old, the ensuing results for vector-valued forms are recent. The second, discussed in §4, is to use modern geometric invariant theory to study the quotient space directly. While the set Herm$/G$ has a standard quotient topology, it does not carry a globally defined differentiable

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