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Deformation Theory of CR-Structures and Its Application to Deformations of Isolated Singularities I

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Introduction

Let (V, o) be a normal isolated singularity in C^N of complex dimension n. We would like to study a deformation theory of complex structures of (V, o). This problem is studied in several ways. For example, (1) Grauert's method (cf. [Gr1]), (2) Douady's method (cf. [Dou]), (3) Kuranishi's approach (cf. [Ku1], [Ku2]), etc. In this paper, we recall Kuranishi's approach and give a review of some contribution, done by T. Akahori and K. Miyajima (cf. [Ku1], [Ku2], [Ak1]-[Ak5], [Ak-My1], [My1]).

Now we set the intersection of V with the real hypersphere centered at o of radius ϵ, namely

$$M = V \cap S_\epsilon^{2N-1}.$$

This M is a non-singular real $2n-1$ dimensional C^∞ manifold, and over this M, a CR structure is induced from V. Namely, ${}^0T'' = C \otimes TM \cap T''N \mid_M$, where $N = V - o$. Conversely, this CR structure $(M, {}^0T'')$ determines the normal Stein space V, uniquely. Noting this result, in order to give a versal family of deformations of singularities, Kuranishi initiated his deformation theory of CR strucutres for a normal isolated singularity. To see Kuranishi's approach and to see our contribution, we recall Kodaira-Spencer's theory for deformation theory of complex structures of compact complex manifolds.

Let X be a complex manifold, and let $(X, T''X)$ denote the complex structure. Then, the deformation theory of complex structures proceeds as follows.

1) *Formulation.* Any deformation of the given complex structure $T''X$, can be parametrized by an element ϕ of $\Gamma(X, T'X \otimes (T''X)^*)$,

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