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Geometry of Matrices

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***In Memory of Professor L. K. Hua* (1910–1985)**

§1. Introduction

The study of the geometry of matrices was initiated by L. K. Hua in the mid forties [5–10]. At first, relating to his study of the theory of functions of several complex variables, he began studying four types of geometry of matrices over the complex field, i.e., geometries of rectangular matrices, symmetric matrices, skew-symmetric matrices, and hermitian matrices. In 1949, he [11] extended his result on the geometry of symmetric matrices over the complex field to any field of characteristic not 2, and in 1951 he [12] extended his result on the geometry of rectangular matrices to any division ring distinct from $\mathbb{F}_2$ and applied it to problems in algebra and geometry. Then the study of the geometry of matrices was succeeded by many mathematicians. In recent years it has also been applied to graph theory.

To explain the problems of the geometry of matrices we are interested in, it is better to start with the Erlangen Program which was formulated by F. Klein in 1872. It says: "A geometry is the set of properties of figures which are invariant under the nonsingular linear transformations of some group". There F. Klein pointed out the intimate relationship between geometry, group, and invariants. Then a fundamental problem in a geometry in the sense of Erlangen Program is to characterize the transformation group of the geometry by as few geometric invariants as possible. The answer to this problem is often called the fundamental theorem of the geometry.

In a geometry of matrices, the points of the associated space are a certain kind of matrices of the same size, and there is a transformation

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