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Towards a Classification of Spin Models in terms of Association Schemes

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§1. Introduction

The *spin models* considered here have been introduced by V. Jones [Jo] (in the symmetric case) and by Kawagoe, Munemasa, Watatani [KMW] (in the general case) as basic data for a certain construction of invariants of links in 3-space. Such a spin model consists in a pair of square matrices satisfying some constraints which we call *invariance equations*. Links are represented by plane diagrams and the matrices of the spin model are used to assign to every such diagram a number (this number is the value of the *partition function*). The invariance equations represent sufficient conditions on the matrices of the spin model which insure that the partition function (multiplied by a suitable normalization factor) is invariant under simple deformations of diagrams called *Reidemeister moves*. These moves describe in terms of diagrams the natural topological equivalence of links, and hence the partition function of every spin model defines a link invariant.

The pioneering work of Jones gave two examples of symmetric spin models and raised the question of finding new ones. It turned out that this question is intimately related with the theory of association schemes. Indeed many subsequent works (in particular [Ja1], [B3], [BB1], [BB2], [BBIK], [BBJ], [BJS], [I1], [I2], [I3], [Ja3], [N1], [N2], [N3]) confirmed the importance of the following situation: the matrices of a spin model belong to the Bose-Mesner algebra of some self-dual association scheme, and can be obtained by solving a certain *modular invariance equation* associated with the (suitably indexed) first eigenmatrix of the scheme (more details can be found in the surveys [B2], [Ja2], [Ja4]). The main purpose of the present paper is to show in the symmetric case that actually this situation is completely general.

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