Advanced Studies in Pure Mathematics 23, 1994
Spectral and Scattering Theory and Applications
pp. 311–322

Asymptotics for the Number of Negative Eigenvalues of Three–Body Schrödinger Operators with Efimov Effect

Hideo Tamura

Introduction

The Efimov effect is one of the most interesting results in the spectral analysis for three–body Schrödinger operators. Roughly speaking, it can be explained as follows: If all three two–body subsystems have no negative eigenvalues and if at least two of these subsystems have a resonance state at zero energy, then the three–body system under consideration has an infinite number of negative eigenvalues accumulating at zero. This remarkable spectral property was first discovered by Efimov [1] and the mathematically rigorous proof has been given by the works [4, 8, 10]. In the present note, we study the asymptotic distribution of these negative eigenvalues below the bottom zero of essential spectrum which is a three–cluster threshold energy. Let $N(E)$, $E > 0$, be the number of negative eigenvalues less than $-E$ with repetition according to their multiplicities. Then the result obtained here is, somewhat loosely stating, that $N(E)$ behaves like $|\log E|$ as $E \to 0$.

We first formulate precisely the main theorem and then make a brief comment on the recent related result obtained by Sobolev [7]. We consider a system of three particles with masses $m_j > 0$, $1 \le j \le 3$, which move in the three–dimensional space R^3 and interact with each other through a pair potential $V_{jk}(r_j - r_k)$, $1 \le j < k \le 3$, where $r_j \in R^3$ denotes the position vector of the j–th particle. For such a system, the energy Hamiltonian H (three–body Schrödinger operator) takes the form

$$(0.1) \qquad H = H_0 + V, \qquad V = \sum_{1 \le j < k \le 3} V_{jk}(r_j - r_k),$$

in the center–of–mass frame, where H_0 denotes the free Hamiltonian. Both the operators H_0 and H act on the space $L^2(R^6)$ and are repre-

Received January 7, 1993.