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Inverse Iteration Method with a Complex Parameter II

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§1. Introduction

Let A be a symmetric (n, n) matrix and let $\lambda_k, \phi_k, k = 1, \ldots, n$ be pairs of eigenvalues and the corresponding eigenvectors of A. The inverse iterarion process for the eigenvector ϕ_j is to solve the following linear equations with initial data $z^{(0)}$ under the conditions $|\lambda_j - \lambda| << c < |\lambda_k - \lambda|, (k \neq j)$:

$$(A - \lambda I)z^{(m+1)} = z^{(m)}, m = 0, 1, 2, \ldots \tag{1.1}$$

In the paper [1] we proposed the inverse iteration method with a complex parameter and showed some numerical results of our method. There we replaced λ in (1.1) by a complex parameter $\lambda + \sqrt{-1}\tau$ and managed to derive the utilities of the complex parameter with $|\tau| < \varepsilon$ under the following Assumption H.

Assumption H. Eigenvalues $\lambda_k, k = 1, 2, \ldots, n$ of A are known with the following accuracy: There are three numerical constants c,ε and λ such that $\inf_{k \neq j} |\lambda_j - \lambda_k| > 2c$ and $|\lambda_j - \lambda| < \varepsilon$ and $0 < 2\varepsilon < c$.

In the spectral theory, it is well known that the projection operator P_j to the eigenspace correspondong to the eigenvalue λ_j is represented as follows

$$P_j v = \frac{1}{2\pi\sqrt{-1}} \oint (A - \zeta I)^{-1} v d\zeta. \tag{1.2}$$

It can be considered that to solve the linear equation (1.1) is to execute the numerical integral of (1.2) with one point value. Since the result of our method is understood to be that with two point values, it will be taken for granted that our method is more effective than the standard traditional one.

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