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Asymptotic Behavior of Solutions for the Coupled Klein-Gordon-Schrödinger Equations

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Dedicated to Professor S.T. Kuroda on his 60th birthday

§1. Introduction and theorem

In the present paper we consider the asymptotic behavior in time of solutions for the coupled Klein-Gordon-Schrödinger equations:

$$(1.1)\quad i\frac{\partial}{\partial t}\psi + \frac{1}{2}\Delta\psi = \phi\psi, \quad t \in \mathbf{R}, \quad x \in \mathbf{R}^N,$$

$$(1.2)\quad \frac{\partial^2}{\partial t^2}\phi - \Delta\phi + \phi = -|\psi|^2, \quad t \in \mathbf{R}, \quad x \in \mathbf{R}^N,$$

$$(1.3)\quad \psi(0,x) = \psi_0(x), \quad \phi(0,x) = \phi_0(x), \quad \frac{\partial}{\partial t}\phi(0,x) = \phi_1(x).$$

Equations (1.1)–(1.2) describe a classical model of Yukawa's interaction of conserved complex nucleon field with neutral real meson field and the associated mass has been normalized as unity. Here ψ is a complex scalar nucleon field, and ϕ is a real scalar meson field. (1.1)–(1.2) are a semi-relativistic version of the coupled Klein-Gordon-Dirac equations (see, e.g., [2]).

Since the interaction above is only quadratic, the problems concerning asymptotic behavior of solutions are harder than the cases of higher interactions, especially in lower space dimensions. In order to examine the basic structure of nonlinearities of (1.1)–(1.2), it would be instructive to look at the decoupled case with self-interaction.

There are a large amount of papers concerning the asymptotic behavior in time of solutions for the nonlinear Schrödinger equation

$$(1.4)\quad i\frac{\partial}{\partial t}u + \frac{1}{2}\Delta u = |u|^{p-1}u, \quad ,t \in \mathbf{R}, \quad x \in \mathbf{R}^N,$$

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