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Sufficient Condition for Non-uniqueness of the Positive Cauchy Problem for Parabolic Equations

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Dedicated to Professor ShigeToshi Kuroda on the occasion of his 60th birthday

§1. Introduction

The purpose of this paper is to give a sufficient condition for non-uniqueness of non-negative solutions of the Cauchy problem

$$(1) \qquad (\partial_t - \Delta + V(x))u(x,t) = 0 \qquad \text{in} \quad R^n \times (0,\infty),$$

$$(2) \qquad u(x,0) = 0 \qquad \text{on} \quad R^n,$$

where V is a real-valued function in $L_{p,\mathrm{loc}}(R^n)$, $p > n/2$ for $n \geq 2$ and $p = 1$ for $n = 1$. We mean by a solution of (1)–(2) a function which belongs to

$$C^0(R^n \times [0,\infty)) \cap L_{2,\mathrm{loc}}([0,\infty); H^1_{\mathrm{loc}}(R^n_x))$$

and satisfies (1) and (2) in the weak sense and continuously, respectively (cf. [A]). We assume that

$$(3) \qquad |V(x) - W(|x|)| \leq C \qquad \text{on} \quad R^n$$

for some constant $C \geq 0$ and a measurable function W on $[0,\infty)$ with $\inf_{r \geq 0} W(r) > 0$. Our main result is the following

Theorem. *Suppose that*

$$(4) \qquad \int_1^\infty W(r)^{-1/2} dr < \infty.$$

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