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Singularities of Solutions to System of Wave Equations with Different Speed

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§1. Introduction and results

We consider the following system of wave equations

$$(1.1) \qquad \begin{cases} \Box_{c_1} u = f(u,v) \\ \Box_{c_2} v = g(u,v) \end{cases}$$

where $\Box_c = (1/c^2)\partial^2/\partial t^2 - \sum_{j=1}^n \partial^2/\partial x_j^2$ and c_1 and c_2 are positive constants. We assume that $f(\cdot,\cdot)$ and $g(\cdot,\cdot)$ are in C^∞. In what follows, we shall study the singularities of the solutions to (1.1) when the solutions are 'conormal distributions' to some hyperplanes. Before the statement of main theorems, we define conormal distributions.

Definition (Conormal distributions). *Let $\Omega \subset \mathbb{R}^n$ be a domain. Let L be a C^∞-manifold in Ω. We call that u is in $H^s(L,\infty)$ in Ω if*

$$M_1 \circ M_2 \circ \cdots \circ M_l u \in H^s_{loc}(\Omega) \quad \textit{for } l = 0,1,2,\dots,$$

where each M_j is a C^∞ vector field which is tangent to L.

We can define the space of conormal distributions not only for a C^∞-manifold but also for a union of two hypersurfaces which intersect each other transversally.

Now we shall state the main results. Let $\omega \in S^{n-1}$ and $L_{ij} = \{(t,x) \in \mathbb{R}^n; c_i t + (-1)^j \omega \cdot x = 0\}$ for i, $j = 1$, 2.

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