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# $H^1$-Blow up Solutions for Peker-Choquard Type Schrödinger Equations

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## §1. Introduction and the main results

In this paper, we study the $H^1$-solution for the following nonlinear Schrödinger equation

$$\text{(1-1)} \qquad \begin{cases} i\partial_t u = -\Delta_x u - (r^{-\gamma} * |u|^2)u \\ u(0,x) = u_0(x) \in H^1(\mathbf{R}^N) \end{cases},$$

where $r = |x|$ and $2 \leq \gamma < 4$, $\gamma \leq N-1$, and show a sufficient condition of '$H^1$-blowing up'. Here we say that $u$ is an $H^1$-local solution of (1-1) when for some $T > 0$, $u \in C([0,T); H^1)$ and satisfies next integral equation

$$\text{(1-2)} \qquad u(t) = U(t)u_0 - i\int_0^t U(t-s)\{(r^{-\gamma} * |u^2|)u\}(s)ds,$$

where $U(t) = \exp(it\Delta_x)$ is the evolution operator for the free Schrödinger equation. Above type nonlinear Schrödinger equation is appeared in some approximations of many body problems, so-called Hartree approximation. As for detailed arguments of this approximation, see e.g. [5], [6] and [7].

Before stating the main results, we define several notations. For $p \in [1,\infty]$ and $k \in \overline{\mathbf{N}}$, we define Sobolev space

$$W^{k,p} \equiv \{f \in \mathcal{S}' : \|f\|_{W^{k,p}} \equiv \sum_{|\alpha|\leq k} \|\partial_x^\alpha f\|_p < \infty\},$$

where $\|\cdot\|_p$ is usual $L^p$-norm. $H^k \equiv W^{k,2}$ and $H^{-k} \equiv (H^k)^*$. For an interval $I$ and a Banach space $X$, $C^k(I;X)$ is the space of $X$-valued $C^k$-functions on $I$, $k = 0,1,2...$ and $L^p(I;X)$ is the space of $L^p$-functions. We say $u \in L^p_{\text{loc}}(I;X)$ if $u \in L^p(J;X)$ for any compact $J \subset I$.

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