Advanced Studies in Pure Mathematics 23, 1994
Spectral and Scattering Theory and Applications
pp. 105–111

The Relativistic Boltzmann Equation Near Equilibrium

Robert T. Glassey and Walter A. Strauss

§1. A remark on spectral theory

The Boltzmann equation, linearized around the equilibrium, has the form

$$\frac{\partial f}{\partial t} + A f + K f = 0.$$

We want to deduce the exponential decay of $f(t)$ as $t \to \infty$. The operator $A + K$ is neither symmetric nor skew-symmetric. Nor is K compact. However, it enjoys the following properties:

(i) Spec $(A + K) \subset \{\text{Re } \lambda \geq 0\}$

(ii) $A + K$ has no point spectrum on Re $\lambda = 0$.

(iii) Spec $(A) \subset \{\text{Re } \lambda \geq \alpha_0\}$ for some $\alpha_0 > 0$.

(iv) K is A -smoothing.

Property (iv) means, roughly, that the operator

$$e^{-t_1 A} K e^{-t_2 A} K \dots e^{-t_\ell A} K$$

is compact for all $t_1 > 0, \dots, t_\ell > 0$.

Theorem [Vidav, Shizuta]. *The spectrum of $A + K$ in the strip $\{0 \leq \text{Re } \lambda < \alpha_0\}$ is discrete, and*

$$\| e^{-t(A+K)} \| \leq e^{-\alpha_1 t}$$

for some $\alpha_1 > 0$.

This is a generalization of Weyl's classical theorem on the perturbation of spectra. We will see at the end of the lecture how this theorem proves the stability of the equilibrium of the relativistic Boltzmann equation.

Received December 4, 1992.
Research supported in part by National Science Foundation grants DMS 90-23864 and DMS 90-23196 and by ARO grant DAAL-3-90-G0012.