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Commutator Algebra and Resolvent Estimates

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§1. Introduction

In studying the detailed properties of Schrödinger operators, the method of micro-localization seems to be indispensable. For the many-body problem, this point of view was introduced by Enss [3], Mourre [11] and then by Sigal-Soffer [13] to investigate the propagation properties of the unitary group. These sorts of estimates not only lead us to a deep understanding of the space-time behavior of the solution to the Schrödinger equation, but also give us many applications. The aim of this paper is to prove a certain variation of these kinds of estimates for the resolvent of the N-body Schrödinger operator.

We consider a system of N-particles moving in $\mathbf{R}^\nu$ with mass m_i and position $x^i \in \mathbf{R}^\nu (1 \le i \le N)$. Let $\mathcal{X}$ be defined by

$$\mathcal{X} = \{(x^1, \cdots, x^N); \sum_{i=1}^N m_i x^i = 0\},$$

and consider the Schrödinger operator

$$H = H_0 + \sum_{i<j} V_{ij},$$

where $-H_0$ is the Laplace-Beltrami operator on $\mathcal{X}$ equipped with the Riemannian metric induced from $ds^2 = 2\sum_{i=1}^N m_i (dx^i)^2$ on $\mathbf{R}^{N\nu}$. Each pair potential $V_{ij} = V_{ij}(x^i - x^j)$ is assumed to be a real-valued C^∞-function on $\mathbf{R}^\nu$ and satisfies for some constant $\rho > 0$

$$|\partial_y^m V_{ij}(y)| \le C_m <y>^{-m-\rho}, \tag{1.1}$$

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