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Tits Metric and Visibility Axiom

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§1. Introduction

An Hadamard manifold H or H^n, i.e., a complete connected simply-connected n-dimensional Riemannian manifold with non-positive sectional curvature is called *a visibility manifold* if the angles at a fixed point subtended by geodesics going far away are arbitrarily small enough no matter how long they are. This condition given by P. Eberlein and B. O'Neill [3] plays basic roles in the study of Hadamard manifolds. They also defined the concept of points at infinity, $H(\infty)$, and it is known that H is a visibility manifold if and only if any different two points at infinity $x_1, x_2 \in H(\infty)$ can be joined by a geodesic of H. This property is called *the axiom* 1. The next two theorems determining this condition are classical:

Theorem 1 ([2],[3]). *If the sectional curvature of H is bounded above by a negative constant, then H is a visibility manifold.*

Theorem 2 ([1]). *In the case of H^n being a surface H^2, it is a visibility surface if and only if for every sector S of H^2, the total curvature of S,*

$$\iint_S K\, dv = -\infty$$

holds, where K is the Gaussian curvature and a sector S is a piece of surface which is cut off by two different rays starting a common point.

Theorem 1 is proved in [2] Lemma 9-10, and also in [3] Proposition 5-9 with an extended form using the idea of curvature order. These proofs in any cases depend essentially on the so-called Gauss-Bonnet theorem on surfaces. Similarly, using the Gauss-Bonnet theorem, we can prove easily Theorem 2 (cf. [1] page 57). Paying attention to the polar coordinate expression around a point in Theorem 2, K. Uesu [5]

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