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Super Lie Groups

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In recent years the theory of super Lie groups has been studied by many authors in different formulations. See [1] for general references. We have developed the theory of super manifolds in previous notes [2] and [3]. With the same formulation used in the latter, here we shall consider super Lie groups and prove some fundamental existence theorems.

§1. Preliminary

In this note we shall basically follow the arguments and notations in [3]. However, we shall make some change in notations so that our arguments will be more coherent with the theory of ordinary Lie groups.

Let M be a super manifold and $\mathcal{O}_z$ the set of all germs of super smooth functions at a point $z \in M$. A super tangent vector at $z \in M$ was defined in [3]. But in this note we define a super tangent vector as follows.

A mapping v of $\mathcal{O}_z$ into Λ whose image of $f \in \mathcal{O}_z$ is written by $v \cdot f \in \Lambda$ is called a *super tangent vector* at $z \in M$ if v satisfies the following conditions: for $f, g \in \mathcal{O}_z$ and $a \in \Lambda$,

$$\begin{aligned} &1) \quad v \cdot (f+g) = v \cdot f + v \cdot g, \\ &2) \quad v \cdot (fa) = (v \cdot f)a, \\ &3) \quad v \cdot (fg) = (v \cdot f)g(z) + (-1)^{fg}(v \cdot g)f(z), \end{aligned}$$

where f, g in $(-1)^{fg}$ denote their parities of f, g. Then the set of all super tangent vectors at $z \in M$ forms a super vector space called the *super tangent space* at $z \in M$, denoted by $T_z(M)$. This change is not at all essential. Actually, this $T_z(M)$ can be identified with the old $T_z(M)$ in [3] in a natural way. See [1] for the details of super linear algebra. When (z^i) is a local coordinate around $z \in M$, $\{(\frac{\vec{\partial}}{\partial z^i})_z\}$ forms a base

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