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Bubbling of Minimizing Sequences for Prescribed Scalar Curvature Problem

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§1. Introduction

Let (M, g) be a compact Riemannian manifold of dimension $n\ (\geq 3)$ and K be a smooth function on M. In this paper we consider the problem of finding a metric conformal to g having the scalar curvature K. Any conformal metric to g can be written $\tilde{g} = u^{2/(n-2)}g$ where u is a positive smooth function on M. From the transformation law for the scalar curvature, this problem is equivalent to solve the nonlinear partial differential equation

$$(1.1)\qquad \begin{gathered} L_g u := -\kappa\Delta_g u + Ru = Ku^{N-1}, \qquad u > 0 \qquad \text{in } M, \\ \kappa = \frac{4(n-1)}{n-2}, \qquad N = \frac{2n}{n-2}, \end{gathered}$$

where Δ_g denotes the negative definite Laplacian and R is the scalar curvature of g. The linear elliptic operator L_g is called the conformal Laplacian of (M, g). In the case K is a constant the problem was first studied by Yamabe [26]. For general K the problem was presented by Kazdan-Warner [16], [17]. Since their pioneer work, the problem has drawn attentions of both geometers and analysts (for example, see [3], [11], [14]).

As proved in [15], the problem can be reduced to the case where scalar curvature R is everywhere either positive, zero or negative. Here we consider only the case that R is positive everywhere. In this case, we easily see that a necessary condition for the solvability of (1.1) is that K is positive somewhere. For such function K, the problem has the variational formulation. We consider the functional

$$E(u) = \int_M (\kappa|\nabla u|^2 + Ru^2)\, dV,$$

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