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The Length Function of Geodesic Parallel Circles

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Dedicated to Professor T. Otsuki on his 75th birthday

§0. Introduction

The isoperimetric inequalities for a simply closed curve C on a Riemannian plane Π (i.e., a complete Riemannian manifold homeomorphic to $\mathbf{R}^2$) was first investigated by Fiala in [1] and later by Hartman in [2]. These inequalities were generalized by the first named author in [3],[4] for a simply closed curve on a finitely connected complete open surface and by both authors in [5] for a simply closed curve on an infinitely connected complete open surface. Here a noncompact complete and open Riemannian 2-manifold M is called *finitely connected* if it is homeomorphic to a compact 2-manifold without boundary from which finitely many points are removed, and otherwise M is called *infinitely connected.* Fiala and Hartman investigated certain properties of geodesic parallel circles $S(t) := \{x \in \Pi\ ;\ d(x, C) = t\}$, $t \geq 0$ around C of a Riemannian plane Π in order to prove the isoperimetric inequalities, where d denotes the Riemannian distance function. Fiala proved in [1] that if a Riemannian plane Π and a simple closed curve C on Π are *analytic*, then $S(t)$ is a finite union of piecewise smooth simple closed curves except for t in a discrete subset of $[0, \infty)$ and its length $L(t)$ is *continuous* on $[0, \infty)$. If Π and C are *not analytic but smooth*, then $L(t)$ is *not always continuous* as pointed out by Hartman in [2]. What is worse is that $S(t)$ does not always admit its length. Under the assumption of low differentiability of Π and C, Hartman proved that $S(t)$ is a finite union of piecewise smooth simple closed curves except for t in a closed subset of Lebesgue measure zero in $[0, \infty)$. This result was recently extended by the authors [5] to an arbitrary given simply closed curve C in an arbitrary given complete, connected, oriented and noncompact Riemannian 2-manifold M.

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