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A Geometric Construction of Laguerre-Forsyth's Canonical Forms of Linear Ordinary Differential Equations

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§0. Introduction

The purpose of this paper is to reformulate the fundamental results of E.J. Wilczynski's book [5] by applying E. Cartan's method of moving frames.

Let E be a vector bundle over a 1-dimensional manifold M. By taking a local coordinate system t in M and a moving frame $\{e_1, \cdots, e_r\}$ of E, we express each cross section s of E in the form:

$$s = \sum_{\alpha=1}^{r} y_\alpha e_\alpha, \tag{0.1}$$

where $y_1, \cdots, y_r$ are functions on M. Let $\mathcal{D}$ be a system of homogeneous linear ordinary differential equations on E of order n given in the form:

$$\left(\frac{d}{dt}\right)^n y_\alpha + \sum_{k=1}^{n}\sum_{\beta=1}^{r} a_{\alpha\beta}^{(k)}(t) \left(\frac{d}{dt}\right)^{n-k} y_\beta = 0, \quad \alpha = 1, \cdots, r. \tag{0.2}$$

Corresponding to (0.2), we define $r \times r$ matrices $A^{(1)}(t), \cdots, A^{(n)}(t)$ by

$$A^{(k)}(t) = (a_{\alpha\beta}^{(k)}(t)), \quad k = 1, \cdots, n. \tag{0.3}$$

For another local coordinate system in M and another moving frame, we also express $\mathcal{D}$ in a similar way as (0.2) and give $r \times r$ matrices as (0.3).

In modern terminology, Wilczynski showed the following facts:

(A) There exists a pair $(t, \{e_\alpha\})$ satisfying the following condition

$$A^{(1)}(t) = 0 \quad \text{and} \quad \mathrm{Tr}\ A^{(2)}(t) = 0. \tag{L.F}$$

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