Advanced Studies in Pure Mathematics 22, 1993
Progress in Differential Geometry
pp. 247–263

On the L^2 Cohomology Groups of Isolated Singularities

Takeo Ohsawa

Dedicated to Professor Noboru Tanaka on his 60th birthday

Introduction

Let (V, x) be a (complex) n-dimensional isolated singularity. Given a Hermitian metric on $V\backslash\{x\}$, say ds^2, the r-th L^2 cohomology group of V at x is defined as the inductive limit of the L^2 de Rham cohomology groups $H^r_{(2)}(U\backslash\{x\}, ds^2)$, where U runs through the neighbourhoods of x. Recently, L. Saper [10] established a remarkable result that there exist Kähler metrics on $V\backslash\{x\}$, complete near x, for which the r-th L^2 cohomology groups of V at x are zero whenever $r \geq n$. It implies an important fact that the intersection cohomology group of a Kähler variety with isolated singularities carries a canonical Hodge structure. Relying on Saper's result, the author could show that the L^2 cohomology vanishing as above is also true with respect to the restriction of the euclidean metric associated to any holomorphic embedding $(V, x) \hookrightarrow (\mathbf{C}^N, 0)$ (cf. [7]). The purpose of the present article is to complement these works by giving a self-contained version of the latter work. Namely we shall first establish an abstract vanishing theorem as a consequence of a new L^2 estimate with respect to a certain family of metrics and weights which seems to be of interest in itself. Then we shall proceed to apply it to prove a vanishing theorem of Saper type with respect to a certain class of complete Kähler metrics which is actually wider than Saper's ones. Hopefully our method will be available to investigate the L^2 cohomology of spaces with non-isolated singularities. Next we shall give a new proof of our previous result mentioned above. The argument here is essentially the same except that we do not appeal to the existence of a projective variety containing (V, x) and tried to make the argument more transparent. Therefore some part of the proof will be only sketchy.

Received February 20, 1991.
Revised June 17, 1991.