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On a Theorem of Edmonds

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§1. Introduction

For an action of a cyclic group of odd order $m \geqq 3$ on a manifold, the normal bundle of the fixed point set is orientable. It is false for actions of the cyclic group of order 2. Edmonds showed the following

Theorem (Edmonds [E]). *If $\mathbb{Z}_2$ acts smoothly on an n dimensional spin manifold M, preserving its orientation and spin structure, then the fixed point set $F = M^{\mathbb{Z}_2}$ is orientable.*

Bott and Taubes gave another proof in [B-T]. The purpose of this short note is to give an elementary proof of this theorem and consider the spin^c case.

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§2. Review on Clifford algebras and spin structures

Let V be a vector space with an positive definite inner product. The Clifford algebra $\mathrm{Cl}(V)$ associated to V is defined as the quotient algebra of the tensor algebra over V by the ideal generated by $v \otimes v + |v|^2$ where $v \in V$. $\mathrm{Cl}(V)$ is not an algebra with $\mathbb{Z}$-grading, but there is a filtration as follows:

$$\mathrm{Cl}(V)^k = \textit{linear span of } \{v_1 \cdots v_j \in \mathrm{Cl}(V); v_i \in V, j \leqq k\}.$$

It is easy to see that the associated graded module of filtered module $\mathrm{Cl}(V)$ is the exterior algebra $\Lambda(V)$. $\mathrm{Cl}(V)$ contains the spin group $\mathrm{Spin}(V)$ which is the double covering group of $SO(V)$. More precisely $\mathrm{Spin}(V) = \mathrm{Pin}(V) \cap \mathrm{Cl}(V)_0$, where $\mathrm{Pin}(V)$ is the multiplicative group generated by unit vectors in V, and $\mathrm{Cl}(V)_0$ is the even part of $\mathrm{Cl}(V)$

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