Advanced Studies in Pure Mathematics 22, 1993
Progress in Differential Geometry
pp. 231–242

Green Function on Self-Similar Trees

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§1. Introduction

There are typical examples of symmetric homogeneous spaces where the canonical Green functions associated with the Laplacians are explicitly calculated [29]. However, it is usually difficult to calculate them on models without nice symmetry or rich group structure.

In this note we shall study one dimensional models which have self-similar structure instead of symmetric one and shall derive a functional equation via a scaling argument which determines in principle the Green function. An asymptotic expansion of the Green function will also be discussed which gives the decay order of the heat kernel as time goes to infinity.

We are partly motivated by fractal geometry. In fact self-similar trees are typical fractal models [19] [13] [11] and the asymptotic decay order of heat kernels is in general closely related to the so called spectral dimension of fractal models [25]. Note also that the tree structure is omnipresent in the natural world [19] [18].

We hope that our approach also enrich the knowledge on the spectral geometry (or differential geometry) and on the brownian motion on various models.

§2. Self-similar tree

Let X be the self-similar tree network depicted as in the following Figure 1.

Let the length of PP′, PQ and PR be respectively 1, r and s. Here the self-similarity means that the lengths of QS, QT, RU and RV are respectively r^2, rs, rs and s^2 and moreover other branches are defined in the same manner.

First we choose coordinate x such that the point O and P corresponds respectively to $x = 0$ and $x = 1/2$. To simplify the notation, Q

Received February 1, 1991.