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A Note on Lie Contact Manifolds

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Dedicated to Professor T. Otsuki on his 75th birthday

§1. Introduction

The classical projective and conformal connections of H.Weyl fit into harmonic theory in the Spencer cohomology of graded Lie algebras in the sense that the curvature forms of such connections are harmonic. These structures are treated systematically by N. Tanaka [T1] as special cases of a structure associated with an $\mathfrak{l}$-system, called later a graded Lie algebra of the first kind. A lucid explanation of this theory is given by T. Ochiai [O] where he rebuilds Tanaka theory using semisimple flat homogeneous spaces as model spaces.

To deal with more general structures such as CR-structure, Tanaka developed the theory to simple graded Lie algebras of contact type [T2] and then to the full class of simple graded Lie algebras [T3]. The argument essentially depends on the generalized prolongation scheme and on the harmonic theory in the refined Spencer cohomology of Lie algebras. The vanishing of certain cohomology group guarantees the existence and uniqueness of *the normal Cartan connection* (= *Tanaka connection*, for short), attached to the equivalence class of the structure. Though the curvature form of Tanaka connection is no more harmonic in general, its harmonic part gives a fundamental system of invariants of the structure.

Going back to the starting point, we know that the study of projective and conformal structures on a manifold has a background of the classical projective and conformal geometry. This reminds us of another classical geometry, Lie's sphere geometry. Then what kind of structure corresponds to this geometry? Why has this object not yet been investigated? H. Sato [S, SY] is probably the first to consider this problem and finds a Lie contact structure, which is a structure on a contact manifold with model space T_1S^n of which transformation group is the

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