

Some Remarks on Fields of 2-Planes on Compact Smooth 4-Manifolds

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§1. Introduction

Throughout this paper by a manifold we mean a compact oriented smooth manifold, and by a field of q -planes on a manifold a nonsingular field of oriented tangent q -planes on it.

It is interesting to observe, as a specific feature in four dimension, that same are the conditions for a 4-manifold to admit the following three different structures:

- (A) a field of 2-planes
- (B) a pseudo-riemannian metric of signature $(+ + - -)$ with the structure group $SO_o(2, 2)$
- (C) a pair of an almost complex structure and an opposite almost complex structure.

On the basis of such an observation, the purpose of this paper is to discuss some particular aspects of geometry of 4-manifolds which admit fields of 2-planes.

The paper contains two main results: Theorems 4-1 and 7-2, and is organized as follows. §2 is a quick survey on the problem of fields of 2-planes on 4-manifolds. In §3, we consider the reduction of the structure group of the tangent bundle of a 4-manifold which admits a field of 2-planes in connection with twistor spaces. One part of a couple of the main results is given in §4, which is concerned with the existence of a riemannian metric invariant both by two kinds of almost complex structures on a 4-manifold with a field of 2-planes. In §5, we review the irreducible decomposition of the curvature tensors on an almost Hermitian 4-manifold. In §6, we give an analogue of the irreducible decomposition of the curvature tensors for an opposite almost Hermitian 4-manifold. In the last section (§7), the other part of our main results is stated, which is concerned with the irreducible decomposition of the curvature

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