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A Uniqueness Result for Minimal Surfaces in S^3

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§1. Introduction

In the study of minimal surfaces, the uniqueness for minimal surfaces bounded by a given contour is an important problem which is not yet solved completely.

The first uniqueness result was proved by Radó [4] for minimal surfaces in $\mathbf{R}^3$. He proved that if a Jordan curve Γ has a one-to-one parallel or central projection onto a convex plane Jordan curve, then Γ bounds a unique minimal disk. The second result is due to Nitsche [3] and states that if the total curvature of an analytic Jordan curve Γ does not exceed 4π, then Γ bounds a unique minimal disk. The third result is due to Tromba [6] and states that if a C^2-Jordan curve Γ is sufficiently closed to a C^2-plane Jordan curve in the C^2-topology, then Γ bounds a unique minimal disk.

For minimal surfaces in other Riemannian manifolds, uniqueness theorems in the three dimensional hemisphere of S^3 were proved by Sakaki [5] and Koiso [2]. Sakaki's result is an analogy of Tromba's uniqueness theorem, and Koiso's is an analogy of Radó's theorem.

In this paper we restrict ourselves to minimal surfaces in S^3 which are "graphs" in some sense (Definition 1.1).

Set $S^3 = \{\mathbf{x} \in \mathbf{R}^4; |\mathbf{x}| = 1\}$. Let Σ be a 2-plane in $\mathbf{R}^4$ containing the origin of $\mathbf{R}^4$. We denote by B the two dimensional unit open disk in Σ which is bounded by $\Sigma \cap S^3$.

Definition 1.1. Let D be a subset of the closed disk $\overline{B}$. A subset M of S^3 is called a "graph" over D if M intersects with each 2-plane containing a point of D which is orthogonal to Σ in $\mathbf{R}^4$ at precisely one point.

Definition 1.2. (1) A minimal surface M in S^3 is a continuous mapping Φ of a two dimensional compact C^∞-manifold R with boundary

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