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Characterization of Images of Radon Transforms

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§0. Introduction

Since F. John [7], the characterization of images of Radon transforms has been one of the main subjects of the theory of Radon transforms. When we recall that the origin of Radon transform was the transform of functions on the 2-sphere by averaging over the great circles, it is rather surprising to find that the characterization of images of Radon transforms on compact symmetric spaces had not been treated until E. Grinberg [4]. There Grinberg showed that the image of Radon transform concerning real or complex Grassmann manifolds can be characterized by an invariant system of differential operators, using the representation theoretical argument. We can see easily that the characterization may also be done by an invariant differential operator of higher order, though Grinberg did not mention it explicitly.

The purpose of this paper is to give another type of characterization, that is, the characterization by an invariant differential operator that takes values in the sections of a vector bundle. The approach by Grinberg used the left action of a group, and ours uses the right action, which lies, in a sense, on the other side with respect to the bi-sided invariant differential operator. We hope our approach will be the first step to fill some vacancy in the theory of invariant differential operators on compact symmetric spaces.

§1. The Radon transform on the sphere

We first consider the case of the standard sphere S^n of radius 1 in the Euclidean space $\mathbf{R}^{n+1}$. A geodesic γ of the sphere S^n is nothing but a great circle, which is determined by a 2-dimensional vector subspace of $\mathbf{R}^{n+1}$. We shall treat the geodesics with their orientation for convenience' sake. The set of oriented geodesics, which we denote by $\operatorname{Geod} S^n$, is the oriented real Grassmann manifold $G_{n+1,2}(\mathbf{R})$.

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