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Maslov Class of an Isotropic Map-Germ Arising from One-Dimensional Symplectic Reduction

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Dedicated to Professor Noboru Tanaka on his 60th birthday

§0. Introduction

Let (M^{2n}, ω) be a symplectic manifold of dimension $2n$ and $N^n \subset M^{2n}$ be a Lagrangian submanifold with singularities. For each regular point x of N, T_xN is a Lagrangian subspace of the sympletic vector space T_xM.

To investigate the local structure of N near a singular point x_0 of N, it is natural to study the behavior of the distribution $\{T_xN \mid x \text{ is a regular point of } N\}$ near x_0. Then we can grasp an invariant of the singularity, which is called the Maslov class in this paper.

In studying the problem of Lagrangian immersion of surfaces to four dimensional symplectic manifolds, Givental' [G] introduced a Lagrangian variety, so called an open Whitney umbrella or an unfolded Whitney umbrella, and investigated some local and global problems. In particular, he calculated the "Maslov index" of an open Whitney umbrella. The main purpose of this paper is to generalize the result of Givental'.

Singular Lagrangian varieties appear typically in the process of symplectic reduction (see §5, [A2] and [I1]).

Note that singular Lagrangian varieties obtained by reduction are parametrized by isotropic mappings.

Originally, the notion of Maslov class (Keller-Maslov-Arnol'd class) stemed from the asymptotic method of linear partial differential equation, representation theory, and symplectic topology ([A1], [GS], [Gr], [Hö], [M], [V], [W]).

Maslov classes represent obstruction for transversality of two Lagrangian subbundles (see §1). Applying this understanding, we define

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