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Gauss Maps of Complete Minimal Surfaces

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§1. Introduction

In 1961, R. Osserman showed that the Gauss map of a complete nonflat minimal surface immersed in $\mathbf{R}^3$ cannot omit a set of positive logarithmic capacity ([16]). Afterwards, F. Xavier proved that the Gauss map of such a surface can omit at most six points ([25]). In 1988, the author has shown that the number of exceptional values of the Gauss map of such a surface is at most four ([6]). Here, the number four is best-possible. Indeed, there are many examples of nonflat complete minimal surfaces in $\mathbf{R}^3$ whose Gauss maps omit four values. Moreover, he revealed some relations between these results and the defect relation in Nevanlinna theory on value distribution of meromorphic functions, and gave some modified defect relation for the Gauss map of such a surface in [8]. Recently, as an improvement of these results, X. Mo and R. Osserman showed that, if the Gauss map of a nonflat complete minimal surface M immersed in $\mathbf{R}^3$ takes on five distinct values only a finite number of times, then M has finite total curvature ([14]).

The author gave also modified defect relations for the Gauss map G of a complete minimal surface immersed in $\mathbf{R}^m$ for the case where G is nondegenerate as a map into $P^{m-1}(\mathbf{C})$ and, as its application, he showed that G can omit at most $m(m+1)/2$ hyperplanes in general position ([9]). Here, the number $m(m+1)/2$ is best-possible for arbitrary odd numbers and some small even numbers ([7]). Recently, M. Ru showed that the "nondegenerate" assumption of the above result can be dropped ([20]). In [10], the author introduced a new definition of modified defect and proved a refined modified defect relation for the Gauss map of complete minimal surfaces possibly with branch points and gave some improvements of the above-mentioned results in [9], [14] and [20].

The purpose of this lecture is to survey the above-mentioned results more precisely and to give the outline of their proofs. We first give

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