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On Adelic Zeta Functions of Prehomogeneous Vector Spaces with a Finitely Many Adelic Open Orbits

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Introduction

The two adelic zeta functions $Z_a(\omega, \Phi)$ and $Z_m(\omega, \Phi)$ for a prehomogeneous vector space (abbrev. P.V.) (G, ρ, V) have no relation in general. For an irreducible case, Professor J. Igusa showed that $Z_a = \tau Z_m$ with some constant τ when $\#(G_A \backslash Y_A) < \infty$ under the condition (HW) where Y is the open G-orbit in V (see Igusa [4]).

In this paper, we shall show that the condition (HW) is not necessary. Moreover, we shall show that the theorem of the same type holds even for simple P.V's and 2-simple P.V.'s of type I. It is known that when $Z_a = \tau Z_m$ holds, we can generalize Iwasawa-Tate Theory for such P.V.'s and we can have many informations (see T. Kimura [11]).

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§1. Basic definitions

Let G be a connected reductive linear algebraic group and $\rho : G \longrightarrow GL(V)$ a rational representation of G with the open dense G-orbit Y. In this case, we call a triplet (G, ρ, V) a prehomogeneous vector space (abbrev. P.V.). The complement S of Y is a Zariski-closed set which is called the singular set of (G, ρ, V). We assume that the isotropy subgroup H of $\rho(G)$ at a point in Y is connected and semisimple. The irreducible components S_i of codimension one

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