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On the Deformations of the Geometric Structures on the Seifert 4-Manifolds

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We call a closed orientable 4-manifold S *a Seifert* 4-*manifold* if S has a structure of a fibered orbifold $\pi: S \to B$ over some 2-orbifold B with general fiber a 2-torus T^2 where the total space S is a nonsingular manifold. In [10], [11] we discussed the relations between them and certain eight geometries in dimension 4 in the sense of Thurston and also gave their topological classification. Here by a geometric structure of S we mean the structure of the form $\Gamma\backslash X$ diffeomorphic to S where X is a 1-connected complete Riemannian 4-manifold and Γ is a discrete subgroup of the group $\mathrm{Isom}^+ X$ of all orientation-preserving isomorphisms of X acting freely on X. The purpose of this paper is to determine the Teichmüller spaces for their geometric structures in the cases when the base orbifolds are either hyperbolic or euclidean (§1 and §2). Our results are parallel to [5], [6] in which the Teichmüller spaces for the geometric structures on the Seifert 3-manifolds were discussed. But a little more arguments are needed for our cases since we should take account of the nontrivial monodromies. In the meanwhile some of the Seifert 4-manifolds have complex structures compatible to their geometric structures ([12]). In these cases we will also give the relations between the Teichmüller spaces and the deformations of the associated complex structures via the Kodaira Spencer maps. In all cases we treat here these maps are surjective but not injective in general (and hence the Teichmüller spaces are not effectively parametrized as families of complex structures §3). Finally in §4 we also give a remark on the moduli spaces for the geometric structures when the base orbifolds are hyperbolic and show that they are defined as Hausdorff spaces whereas, as is well known, the moduli spaces for the complex structures can not be defined as Hausdorff spaces in general. For simplicity in this paper we only consider the Seifert 4-manifolds over the closed orientable base orbifolds. All the subjects will be considered in the smooth category.

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