

## Deformation Spaces on Geometric Structures

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### 0. Introduction

In this note we shall study geometric structures on smooth manifolds and deformation spaces. In 1981 Thurston gave a lecture on projective structures on surfaces in which he has established the following structure theorems (unpublished):

- I. There is a canonical decomposition by convex hulls on a hyperbolic surface  $S$  which admits a (one dimensional complex) projective structure.
- II. There is an isomorphism between the deformation space  $\mathbf{CP}^1(S_g)$  and the product  $\mathcal{T}(S_g) \times \mathcal{ML}(S_g)$ .

Here  $\mathcal{T}(S_g)$  is the Teichmüller space of a closed orientable surface  $S_g$  of genus  $g \geq 2$  and  $\mathcal{ML}(S_g)$  is the space of measured laminations.

Since there was considerable interest in the argument of proof and the key idea seems to be generalized in higher dimension, we have decided to write down an exposition of the above structure theorems (I), (II).

(Complex) projective structure on surfaces is equivalent to conformally flat structure on surfaces when we identify  $\mathbf{CP}^1 = S^2$  and