Advanced Studies in Pure Mathematics 20, 1992
Aspects of Low Dimensional Manifolds
pp. 167–261

Foundations of Flat Conformal Structure

Shigenori Matsumoto

Dedicated to Professor Masahisa Adachi

Introduction

A flat conformal structure on an n-dimensional manifold N is a maximal system of local charts taking values on S^n, with transition functions Moebius transformations. In short it is a geometric structure modelled on $(\mathcal{M}(S^n), S^n)$, where $\mathcal{M}(S^n)$ denotes the group of Moebius transformations on S^n. Equivalently, it is a conformal equivalence class of conformally flat Riemannian metrics on N if $n \geq 3$. See §1 for Liouville's theorem. By certain abuse we denote a flat conformal structure by the same letter as the underlying manifold.

In dimension 2, flat conformal structures are usually called projective structures and have been extensively studied by various authors in the field of function theory. Analytic methods such as the theory of quasiconformal maps often play crucial roles there. In dimension ≥ 3, however, the situation is quite different. Topology, instead of analysis, provides major tools of study.

The concept of flat conformal structures was first introduced by Kuiper ([35],[36],[37]) around 1950. Thereafter it had been forgotten for some time, until it was revived by Kulkarni ([40],[41],[42],[43]), related with his study of discrete group actions in general. Then came an important turning point when Fried ([13]) established a remarkable theorem concerning closed similarity manifolds. It solved a fundamental and annoying problem which one encounters in the primary stage of the theory, thereby making it possible to have a good grip on elementary flat conformal structures, with Goldman ([15]) and Kamishima ([25]) contributing significantly to this direction.

Received July 9, 1990.
Revised June 13, 1991.