Advanced Studies in Pure Mathematics 20, 1992
Aspects of Low Dimensional Manifolds
pp. 147–166

Invariants of Spatial Graphs

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§1. Introduction

The purpose of this paper is to construct invariants of spatial graphs from regular isotopy invariants of non-oriented link diagrams of *knit trace type*. Kauffman's bracket polynomial [4], which is a version of the Jones polynomial, is of knit trace type. The Dubrovnik polynomial [5], which is used in the definition of the Kauffman polynomial, is also of knit trace type [6]. Hence these two invariants are generalized to invariants of spatial graphs by our method. The Yamada polynomial introduced in [10] is the non-trivial simplest one of our invariants. A similar invariants are introduced in [9] for ribbon graphs. They use quasi-triangular Hopf algebras. But we use representations of knit semigroups or braid groups instead of Hopf algebras.

To introduce regular isotopy invariants of link diagrams of *knit trace type*, we need notion of a *Markov knit sequence*. Let $\mathbb{C}$ be the field of complex numbers. Knit semigroups K_n, $(n = 1, 2, \cdots)$ are introduced in [6] defined by the following generators and relations.

$$\begin{aligned} K_n = \langle \tau_1, \cdots, \tau_{n-1}, \ \tau_1^{-1}, \cdots, \tau_{n-1}^{-1}, \ \varepsilon_1, \cdots, \varepsilon_{n-1} \quad | \\ \tau_i \tau_i^{-1} = \tau_i^{-1} \tau_i = 1, \quad \tau_i \tau_j = \tau_j \tau_i \ (|i-j| \geq 2), \\ \tau_i \tau_{i+1} \tau_i = \tau_{i+1} \tau_i \tau_{i+1}, \quad \tau_i \varepsilon_j = \varepsilon_j \tau_i \ (|i-j| \geq 2), \\ \varepsilon_i \varepsilon_{i\pm1} \varepsilon_i == \varepsilon_i, \quad \varepsilon_i \varepsilon_j = \varepsilon_j \varepsilon_i \ (|i-j| \geq 2), \\ \varepsilon_i \tau_{i\pm1} = \varepsilon_i \varepsilon_{i\pm1} \tau_i^{-1}, \quad \varepsilon_i \tau_{i\pm1}^{-1} = \varepsilon_i \varepsilon_{i\pm1} \tau_i, \\ \tau_{i\pm1} \varepsilon_i = \tau_i^{-1} \varepsilon_{i\pm1} \varepsilon_i, \quad \tau_{i\pm1}^{-1} \varepsilon_i = \tau_i \varepsilon_{i\pm1} \varepsilon_i \rangle \end{aligned}$$

The generators of K_n are presented graphically as in Figure 1. In the graphical presentation, the product of two elements of K_n corresponds to the composite of two diagrams as in the case of braid groups. Let

Received July 6, 1991.