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Behavior of Knots under Twisting

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§1. Introduction

This paper is a continuation of [6] in the study of the twist move of knots. First we recall some notations. Let K be an unoriented smooth knot in the oriented 3-sphere S^3, and V a solid torus endowed with a preferred framing which contains K in its interior and satisfies $w_V(K) \geq 2$. ($w_V(K)$ denotes the geometric intersection number of K and a meridian disk of V.) Let f_n be an orientation preserving homeomorphism of V satisfying f_n(meridian) = (meridian) and f_n(longitude) = (longitude) + n(meridian) in $H_1(\partial V)$. (We shall not distinguish notationally between a homeomorphism and an isomorphism on a homology group induced by it.) We denote the knot $f_n(K)$ in S^3 by $K_{V,n}$. If there exsists an orientation preserving homeomorphism of S^3 carrying K_1 to K_2, then we write $K_1 \cong K_2$. Note that $K_1 \cong K_2$ is the same as saying that K_1 and K_2 are ambient isotopic in S^3. We note that for a given knot K, a solid torus V and an integer n determine a unique knot type. For a given knot K, we have an abundant solid tori which contain K to carry out a twist move. Sect.2 is directed towards the following question : for a given knot K, is it possible to obtain the same knot by twistings along distinct solid tori from K? Concerning the case when an original knot is trivial, we give Example 2.1 and Theorem 2.2. In the case when both solid tori are knotted, we shall give Theorem 2.6 and Examples (see Figures 4, 5). In Sect.3, the behavior of Gromov invariants under twistings will be studied. In Sect.4, we study the effects of twistings on primeness of knots. Throughout this paper $N(X)$, ∂X and $\operatorname{int} X$ denote the tubular neighborhood of X, the boundary of X and the interior of X respectively.

§2. On twistings along distinct solid tori

Let V_1 and V_2 be solid tori containing a knot K. We write $V_1 \cong V_2$ provided that there exists an orientation preserving homeomorphism f

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