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Polyhedral Decomposition of Hyperbolic 3-Manifolds with Totally Geodesic Boundary

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Dedicated to Professor Kunio Murasugi on his sixtieth birthday

§1. Introduction

A hyperbolic manifold will be a riemannian manifold with constant sectional curvature -1. It is shown by Epstein and Penner [1] that every noncompact complete hyperbolic manifold of finite volume, hence having cusps, is decomposed by ideal polyhedra. The decomposition supplies a quite convenient block to study several geometries of the cusped manifold especially in dimension three. See [4] for instance.

A variant of the construction by Epstein and Penner would establish a decomposition of a compact hyperbolic manifold with nonempty geodesic boundary by truncated polyhedra as well, which we plan to discuss in a forthcoming paper [3]. However the process will be rather unseen in the manifold.

In this paper, taking advantage of working only in dimension three, we give a more visible construction of this decomposition. In fact we directly show

Theorem. *Let N be a compact hyperbolic 3-manifold with nonempty totally geodesic boundary. Then the topological decomposition of N dual to the cut locus of ∂N modulo boundary is homotopic by straightening to a polyhedral decomposition.*

The visible process is expected to lead us to the deep understanding of geometry of those manifolds. We apply it for example to find the minimum of their volumes in [2].

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