Advanced Studies in Pure Mathematics 19, 1989
Integrable Systems in Quantum Field Theory and Statistical Mechanics
pp. 417–434

$\mathcal{D}$-Modules and Nonlinear Systems

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§1. Tschirnhaus transformations for algebraic systems

From the start of my research in analysis, I made some programs about how to organize such concepts like functions, generalized functions, differential equations, both linear and nonlinear, and all about that. My first systematic talk about this subject was given in july 1960. Since then, as is well known, much progress was made, at least, in the field of a general theory of linear partial differential equations by means of the concept of $\mathcal{D}$-modules and specialized concept of holonomic systems. They have several applications made by Kashiwara, Kawai and others. But my original program was just to develop the theory of nonlinear equations in the same spirit. So I shall give a brief sketch about it.

First recall the special case of algebraic geometry, that is, the concept of manifolds, vector bundles and things like that which live on a manifold. All these things are presented and studied systematically by means of a commutative ring and modules over it. As already pointed out by René Descartes, geometrical objects like curves, surfaces and others are described by means of algebraic equations like

$$f_i(x) = 0, \qquad 0 \le i \le n,$$

where x denotes points of the ambient linear space. In particular in the case of one variable we have the equation

$$f(x) = 0. \tag{1}$$

This is an algebraic equation in one indeterminate. Many studies were done for such equations in the past. Especially a first systematic study was made by Tschirnhaus in 17-th century. He is a friend of Spinoza. He

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Received January 30, 1989.

*Lecture delivered at RIMS, Kyoto, October 18, 1988. Notes taken by A. Nakayashiki.