Advanced Studies in Pure Mathematics 19, 1989
Integrable Systems in Quantum Field Theory and Statistical Mechanics
pp. 287–346

From the Harmonic Oscillator to the A-D-E Classification of Conformal Models

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§1. Introduction

There is a growing feeling that arithmetics will play a significant role in understanding various physical models. This is not a recent discovery. Arithmetic questions arise in a large array of problems ranging from crystallography to complex dynamical systems, from the quantization conditions to applications of group theory and, since this is the subject of the present meeting, in the beautiful interplay between solvable two dimensional statistical models and conformal field theory. This goes perhaps against the opinion that the "queen of mathematics" is to be safeguarded from any application. On the other hand, some physicists have recently coined the name "recreational mathematics" to describe these matters. Let the reader decide if this is an appropriate characterization.

In this contribution I would like to exhibit a few simple examples drawn from work with several collaborators, J. M. Luck and E. Aurell for the first two, A. Cappelli and J.-B. Zuber for the next one and more recently M. Bauer. It is a nice opportunity to thank them warmly again, as well as the organizers of the Taniguchi meeting in Kyoto and my colleagues at the Physics Department of the University of Tokyo for their kind hospitality.

§2. The harmonic oscillator

The harmonic oscillator is one of the simplest integrable systems with energy levels

$$\epsilon_n = \epsilon_0(2n+1) \qquad n \text{ non negative integer} \tag{2.1}$$

in an appropriate scale. Consider a gas of non interacting fermions allowed to occupy these levels, at equilibrium at temperature T and

Received January 9, 1989.