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Paths, Maya Diagrams and representations of $\widehat{\mathfrak{sl}}(r, \mathbf{C})$

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Dedicated to Professor Tosihusa Kimura on his 60th birthday

§1. Introduction

Let $\mathfrak{g}$ be the affine Lie algebra $\widehat{\mathfrak{sl}}(r, \mathbf{C})$, let Λ be a dominant integral weight, and let $L(\Lambda)$ be the irreducible $\mathfrak{g}$-module with highest weight Λ. In this article we construct an explicit basis of each weight space $L(\Lambda)_\mu$. As a corollary we prove a new combinatorial formula for the dimensionality of $L(\Lambda)_\mu$, which was conjectured in [1] through the study of corner transfer matrices of solvable lattice models (see Theorem 1.2 below).

The problem of constructing explicit bases goes back to the work of Gelfand and Tsetlin [2] who gave a canonical basis of $L(\Lambda)$ for the classical Lie algebras $\mathfrak{g} = \mathfrak{gl}(r, \mathbf{C}),\ \mathfrak{o}(r, \mathbf{C})$. Analogous results are available in the setting of affine Lie algebras. When Λ is of level 1, $L(\Lambda)$ can be identified with a space of polynomials in infinitely many variables [3,4] or a simple modification thereof [5]. For higher levels, the Z-algebra approach initiated by Lepowsky and Wilson [6] provides a basis in various cases ($\mathfrak{g} = \widehat{\mathfrak{sl}}(2, \mathbf{C})$, arbitrary levels [3],[7], or $\mathfrak{g} = \widehat{\mathfrak{gl}}(r, \mathbf{C}), \widehat{\mathfrak{sp}}(r, \mathbf{C})$, level 2 [8]). Lakshmibai and Seshadri [9] gave a 'standard monomial basis' for $\widehat{\mathfrak{sl}}(2, \mathbf{C})$ using geometric ideas.

A new feature of our approach is the use of an object—*path*, which we now explain. Let $\epsilon_\mu = (0, \cdots, \overset{\mu-\text{th}}{1}, \cdots, 0)\, (0 \le \mu < r)$ denote the standard base vectors of $\mathbf{Z}^r$. We extend the suffixes to $\mathbf{Z}$ by $\epsilon_{\mu+r} = \epsilon_\mu$. Fix a positive integer l.

Definition 1.1. A path is a sequence $\eta = \left(\eta(k)\right)_{k \ge 0}$ consisting of elements $\eta(k) \in \mathbf{Z}^r$ of the form $\epsilon_{\mu_1(k)} + \cdots + \epsilon_{\mu_l(k)}$ $(\mu_1(k), \cdots, \mu_l(k) \in \mathbf{Z})$.

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