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Solving Models in Statistical Mechanics

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One of the main aims of statistical mechanics is to calculate the partition function Z. Here I shall discuss how this can be done for a certain class of two-dimensional lattice models (and one three-dimensional model). They are by definition "solvable". Most of them can also be related to one-dimensional integrable Hamiltonians, so in this sense they are also said to be "integrable".

Such models are made by placing spins σ_i on the N sites (or edges) of a planar lattice $\mathcal{L}$ (e.g. the square lattice). They have values $+1$ or -1; or $1, \ldots, q$; or indeed any set of values that is appropriate. Adjacent spins (i.e. those sharing an edge, or a face, or a vertex) interact. The partition function is

$$Z = \sum_{\sigma} \prod W(\sigma_i, \sigma_j, \ldots), \tag{1}$$

where the inner product is over all edges, faces or vertices of $\mathcal{L}$; $\sigma_i, \sigma_j, \ldots$ are the spins on each such edge, face or vertex; the sum is over all values of all the spins. If each spin takes q values, there are q^N terms in the summation. We want N to be large: at least 100, and of course q at least 2. Hence there are vastly many terms in the sum.

There are now a number of such solvable models. I list the ones I shall consider here in Table 1. There are of course many others, for instance the Izergin-Korepin [1], nested Bethe ansatz ([2] and refs. therein), and various colouring problems [3–6].

There are many relations between these models: for instance the Ising model [7] is a special case of both the 8-vertex [8] and chiral Potts [9–12] models. The 8-vertex model is equivalent to the 8-vertex solid-on-solid (SOS) model [13], in the sense that they both have the same partition function, even though they are formulated differently and have different order parameters. The hard hexagon model [14] is a special case of the 8-vertex SOS model, and further generalizations of these models

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