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Compact Ricci-Flat Kähler Manifolds

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In this part, we survey general results on compact Kähler manifolds M with $c_1(M)_{\mathbb{R}} = 0$. According to the solution of the Calabi conjecture by Yau [Ya], such a compact Kähler manifold M admits a unique Ricci flat Kähler metric with given Kähler class. Our main interests here are applications of the existence of Einstein-Kähler metrics to studies on topological or holomorphic structures of compact Kähler manifolds M with $c_1(M)_{\mathbb{R}} = 0$.

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§1. Bogomolov decomposition

There are three fundamental types of compact Kähler manifolds whose real first Chern classes vanish:

(1) complex tori T;
(2) *symplectic* Kähler manifolds, i.e., compact Kähler manifolds X of even dimension $2m$ which have a holomorphic 2-form φ with φ^m nowhere vanishing on X (such φ is called holomorphic *symplectic* 2-form);
(3) *special unitary* Kähler manifolds, i.e., compact Kähler manifolds Y of dimension $n \geq 3$ such that the canonical bundle of Y is trivial but $H^0(Y, \Omega^p) = 0$ for $0 < p < n$.

Some examples of compact symplectic Kähler manifolds are given in Section 5. These three types are fundamental in the sense that the following holds:

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