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Einstein Kähler Metrics of Negative Ricci Curvature on Open Kähler Manifolds

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Introduction

In 1954 E. Calabi [C1,2,3] posed a conjecture, now so called Calabi's conjecture. It states that if a real closed $(1,1)$-form γ represents 2π times the first Chern class $c_1(X)$ of a compact Kähler manifolds X, then one can find a Kähler metric whose Ricci form coincides with γ. And an altered version of the conjecture says that a compact Kähler manifold X with negative first Chern class $c_1(X) < 0$ admits an Einstein Kähler metric.

In 1976–77 T. Aubin [A1,2] and S.-T. Yau [Y1,2] proved the conjecture and rich applications to algebraic geometry followed. One of the most remarkable consequences is the Miyaoka-Yau inequality (cf. [M1,2] [Y2]).

Theorem 0.1. *Let X be an n-dimensional compact Kähler manifold with negative first Chern class, then the following inequality between Chern numbers holds:*

$$(-1)^n 2(n+1)c_2(X)c_1(X)^{n-2} \geq (-1)^n n c_1(X)^n.$$

Moreover the equality holds if and only if X is covered by a unit ball in $\mathbf{C}^n$.

It is very natural to try to generalize the existence theorem of Einstein Kähler metrics and the Miyaoka-Yau inequality. Actually Yau himself treated degenerated cases in his paper [Y3] and made an announcement of his extended results in a lecture note of Séminaire Palaiseau 1978 [Y4]. Later on, he and S.-Y. Cheng [C-Y2] showed the existence of Einstein Kähler metrics on bounded domains of holomorphy in $\mathbf{C}^n$, where they introduced the useful notion of bounded geometry. Armed

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