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On Tangent Sheaves of Minimal Varieties

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In this chapter, we shall study the tangent sheaves of minimal varieties, especially concentrating on the stability and an inequality between Chern numbers of the sheaves. To begin with, we shall recall the history of inequality between Chern numbers of minimal varieties.

Van de Ven [V] recognized, for the first time, the inequality $c_1(S)^2 \leq 8c_2(S)$ for a surface of general type S. And he also conjectured that a surface of general type S should satisfy the inequality $c_1(S)^2 \leq 3c_2(S)$. Later, Bogomolov and Gieseker ([Bo], [G]) proved an inequality between Chern numbers:

$$c_1(E)^2 \leq \frac{2r}{r-1}c_2(E)$$

for an H-semistable vector bundle with rank r on a projective surface S, where H is an ample divisor on S. Also Bogomolov obtained an inequality $c_1(S)^2 \leq 4c_2(S)$ for a surface of general type S. Afterward Miyaoka [Mi-1] finally proved Van de Ven's conjecture.

On the other hand, by virtue of Mumford-Mehta-Ramanathan's results ([M-R]), one can easily see that, for an H-semistable vector bundle with rank r on an n dimensional non-singular projective variety M, the following inequality holds:

$$\left\{(r-1)c_1(E)^2 - 2rc_2(E)\right\} \cdot H^{n-2} \leq 0. \tag{0.1}$$

Related to this, Lübke [Lü] proved

$$\int_M \{(r-1)c_1(E) - 2rc_2(E)\} \wedge \Phi^{n-2} \leq 0 \tag{0.2}$$

for an Einstein-Hermitian vector bundle $\{E, h\}$ over an n dimensional Kähler manifold (M, Φ). Now one may ask the relation between stability of vector bundles and Einstein-Hermitian metric. In fact, S. Kobayashi

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