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Compactification of Moduli Spaces of Einstein-Hermitian Connections for Null-Correlation Bundles

Takashi Nitta

§0. Introduction

In 1970's by an effective use of twistor theory originated from Penrose [P], gauge-theoretic studies of anti-self-dual connections over 4-manifolds were inaugurated by Atiyah, Hitchin and Singer (see for instance [A-H-S], [A-J], [A-W]). Almost at the same time, Hartshorne determined the moduli spaces of anti-self-dual connections for SU(2)-bundles over S^4 through a purely algebraic study of the null-correlation bundles over $\mathbb{P}^3(\mathbb{C})$. A little later, Kobayashi [K] introduced the concept of Einstein-Hermitian vector bundles over Kähler manifolds, which is in some sense a higher dimensional analogue of anti-self-dual connections over 4-manifolds (see for instance Kobayashi [K] for a general theory of Einstein-Hermitian connections).

The purpose of this paper is to construct a compactified family of Einstein-Hermitian connections on null-correlation bundles over odd-dimensional complex projective spaces $\mathbb{P}^{2m+1}(\mathbb{C})$. Let $\mathbb{P}^m(\mathbb{H}) = \mathrm{Sp}(m+1)/\mathrm{Sp}(m) \times \mathrm{Sp}(1)$ be the m-dimensional quaternionic projective space, and $p : \mathbb{P}^{2m+1}(\mathbb{C}) \to \mathbb{P}^m(\mathbb{H})$ the corresponding twistor space. The homogeneous space $\mathrm{Sp}(m+1)/\mathrm{id} \times \mathrm{Sp}(1)$ is a principal fibre bundle over $\mathbb{P}^m(\mathbb{H})$ with typical fibre $\mathrm{Sp}(m)$. Let τ be the standard representation of $\mathrm{Sp}(m)$ in $\mathbb{C}^{2m}$. Then $V := (\mathrm{Sp}(m+1)/\mathrm{id} \times \mathrm{Sp}(1)) \times_\tau \mathbb{C}^{2m}$ is a complex vector bundle over $\mathbb{P}^m(\mathbb{H})$. Since $\mathrm{Sp}(m)$ is contained on $\mathrm{U}(2m)$, the vector bundle V carries a natural Hermitian metric h_V. Salamon introduced in [S] a certain type of connections (which we call B_2-connections) on vector bundles over quaternionic Kähler manifolds, and such connections are later studied by Berard-Bergery and Ochiai [B-O] in a more general setting. We showed that B_2-connections are Yang-Mills connections and studied them in [N1], which is also obtained by Capria and

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