Compactification of the Moduli Space of Einstein-Kähler Orbifolds

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Dedicated to Professor Ichiro Satake on his sixtieth birthday

Introduction

In the study of degenerations of various geometric objects such as Einstein metrics or complex structures, the construction of natural compactifications of the corresponding moduli spaces is of crucial importance. For instance, the Satake compactification [17] of the moduli spaces of principally polarized abelian varieties plays a beautiful role in the study of modular forms, while the recent study of Donaldson [8] on the ends of moduli spaces of anti-self-dual connections provides us with new aspects of differentiable 4-manifolds.

The purpose of this paper is to give a natural compactification of the moduli space of polarized Einstein-Kähler orbifolds with a given Hilbert polynomial. We shall then show that, for compact Riemann surfaces, our compactification coincides with those of Mumford [15] and Bers [5] (see 2.6 and 2.7). A couple of other examples of our compactification will be given in Section 4 where we discuss the moduli spaces of polarized Abelian varieties and also of a special type of del Pezzo surfaces (see [12]). As to Abelian varieties, for instance, our approach has some relation to Igusa's compactification [11] in view of both heavy dependence on theta functions. We would also say that Anderson [1], Bando, Kasue and Nakajima [3] recently succeeded in applying Gromov's theory to compactifying the moduli spaces of Einstein manifolds of $\dim_{\mathbb{R}} \leq 4$ with a fixed volume and bounded diameters, though our compactification needs no boundedness of diameters because of the algebro-geometric nature of our construction.

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