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On Rotationally Symmetric Hamilton's Equation for Kähler-Einstein Metrics

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§0. Introduction

R.S. Hamilton [4] proved that any riemannian metric g_0 with positive Ricci curvature on a compact 3–dimensional manifold is deformed to an Einstein metric along the equation

$$\text{(H)} \qquad \frac{d}{dt} g_t = -r_t + \frac{1}{n} \overline{s}_t \cdot g_t \qquad (n = \text{dimension} = 3),$$

where r_t denotes the Ricci tensor of g_t and $\overline{s}_t$ the mean value of the scalar curvature. It is weakly generalized to higher dimensional cases (e.g. [8]).

Equation (H) has good properties: if the initial riemannian metric g_0 is invariant under a group action, then so is each g_t; if g_0 is a Kähler metric, then so is each g_t. In fact, H.D. Cao [2] proves that any Kähler metric on a compact Kähler manifold with vanishing or negative first Chern class is deformed to a Kähler-Einstein metric along equation (H).

This result suggests that, even on a compact Kähler manifold with positive first Chern class, the solution of equation (H) converges to a Kähler-Einstein metric if it exists. The first purpose of this paper is to show that it is true in some special cases given in Y. Sakane [9] and N. Koiso-Y. Sakane [6], [7], which contain rotationally symmetric metrics on the 2-dimensional sphere. On the other hand, if the manifold admits no Kähler-Einstein metrics, then the solution of equation (H) can not converge. But it is interesting to see the behaviour of the solution, which is the second purpose. These situations are unified as the following

Theorem. *Let* $(\widehat{L}, \tilde{g}_0)$ *be a Kähler manifold as in* Chapter VI *and* $\tilde{g}_t$ *be the solution of equation* (H). *Then there is a holomorphic vector*

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