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An Application of Kähler-Einstein Metrics to Singularities of Plane Curves

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Throughout this note, we use the same terminology as [K1] and [K-N-S].

It is a classical problem which kind of singularities a plane curve of degree d can admit. Several authors studied this problem. Among them, Hirzebruch [H], Ivinskis [I] and Yoshihara [Yh] obtained some partial answers under the restriction on the degree d of the given curve. They needed the restriction on d because they used the covering method and the Miyaoka-Yau inequality [M] for surfaces with isolated quotient singularities.

In this note we apply a new differential geometric method to this problem and estimate the maximum number of certain class of singularities of a curve of degree d. Namely, we consider the given curve as a formal branch curve and use the canonical Kähler-Einstein metric on log-canonical normal surfaces with branch loci ([K1], [K-N-S]). Then a general Miyaoka-Yau inequality ([K1], [K-N-S]) gives an estimate for the number of the singularities. Since the Kähler-Einstein metric exists with arbitrarily prescribed formal branch indices, our method has an advantage that we need no covering trick and so need no assumption on d.

Recently, Sakai [S] studies this problem systematically and compares the effectiveness of the estimate obtained by our method and those obtained by other various methods.

Let C be a plane curve of degree d and $C = C_1 + \cdots + C_r$ the decomposition into irreducible components. Let d_i denote the degree of C_i. To each curve C_i, we assign a formal branch index b_i ($2 \leq b_i \leq \infty$ for all i) and make a Q-divisor

$$D = \sum_{i=1}^{r}(1 - \frac{1}{b_i})C_i. \tag{1}$$

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