

Homogeneous Einstein Metrics On Certain Kähler C-Spaces

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*Dedicated to Professor Shingo Murakami
on his 60th birthday*

§0. Introduction

Most known non-standard examples of compact homogeneous Einstein manifolds are constructed via Riemannian submersions. Here the word “standard” means that the Einstein metric on a homogeneous manifold is constructed from the irreducible isotropy representation of the homogeneous manifold. However, such a method does not work effectively if the isotropy representation associated with the homogeneous manifold decomposes into more than two irreducible representations. In fact, only few examples (cf. Wang [8]) of such homogeneous Einstein manifolds are known so far.

Let $M = G/K$ be a Kähler C-space, where G is a compact connected simple Lie group. Then M carries a complex structure J and a Kähler metric g , with respect to J , such that the group $\text{Aut}(M, J, g)$ of holomorphic isometries of the Kähler manifold (M, J, g) acts transitively on M . Assuming now that the associated isotropy representation of K decomposes into non-equivalent three irreducible components, we construct in § 2 examples of such Kähler C-spaces. In § 3, in view of the method of Wang and Ziller (cf. § 1), we find all G -invariant Einstein metrics on the Kähler C-spaces G/K constructed in the preceding section. On the other hand, given a G -invariant complex structure on G/K , we have a unique G -invariant Einstein-Kähler metric on G/K up to homotheties (cf. § 2). Thus if a G -invariant Einstein metric on G/K found in § 3 is Kähler with respect to some G -invariant complex structure on G/K , then it is nothing but a known metric. Therefore, we check in § 4 whether the G -invariant Einstein metrics found in § 3

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