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Harmonic Functions with Growth Conditions on a Manifold of Asymptotically Nonnegative Curvature II

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§0. Introduction

According to a theorem due to Greene-Wu [13], a complete connected noncompact Riemannian manifold M abounds harmonic functions so that M can be imbedded properly into some Euclidean space by them. However various problems on harmonic functions on M with specific conditions (e.g., boundedness, positivity, L^p integrability, etc.) arise in connection with the geometry of M and in fact they have been investigated by many authors (cf. e.g., [11: Section 11], [23], [29: Section 4,6.4] and the references therein). In the previous paper [21], we have discussed bounded or positive harmonic functions on a manifold of asymptotically nonnegative curvature (which will be defined later), and extended all of the results by Li-Tam [24;25] to such manifolds. The purpose of the present paper is to study harmonic functions with finite growth on a manifold of asymptotically nonnegative curvature and then to verify the results stated in [21] without proofs. To state the main results of the paper, we need some definitions. For a harmonic function h on a complete connected noncompact Riemannian manifold M, we denote by $m_x(h,t)$ the maximum of $| h |$ on the metric sphere $S_t(x)$ around a point x with radius t. In this note, h is said to be *of finite growth,* if $\limsup_{t\to\infty} m_x(h,t)/t^p$ is finite for some constant $p > 0$. After Abresch [1], we call M *a manifold of asymptotically nonnegative curvature,* if the sectional curvature K_M of M satisfies:

$$K_M \geq -K \circ r, \qquad \text{(H.1)}$$

where r denotes the distance to a fixed point, say o, of M and $k(t)$ is a nonnegative, monotone nonincreasing continuous function on $[0,\infty)$ such that *the integral* $\int^\infty tk(t)dt$ *is finite.* In [19], we have constructed

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